profile (g). It is also observed that the velocity profiles (f', s') at any two values of λ cross each other towards the edge of the boundary layer. A similar trend has been observed by Yang [11] for the unsteady, two-dimensional, stagnation-point flow over a stationary wall.

4. CONCLUSIONS

The effects of the unsteadiness in the wall velocities and the nature of the stagnation point on the skin friction, heat transfer and mass flux of diffusing species are found to be appreciable. The Prandtl number and the Schmidt number strongly affect the heat transfer and mass flux of diffusing species, respectively. The velocity temperature and concentration profiles are observed to decay exponentially.

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Int. J. Heat Mass Transfer. Vol. 29, No. 12, pp. 1999–2002, 1986
Printed in Great Britain

0017–9310/86 \$3.00+0.00
Pergamon Journals Ltd.

Heating or evaporation in the thermal entrance region of a non-Newtonian, laminar, falling liquid film

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(*Received* 7 *October* 1985 *and in final form* 11 *April* 1985)

1. INTRODUCTION

HEATING or evaporation of non-Newtonian solutions by means of falling film shell-and-tube heat exchangers is sometimes practised in the food and polymer processing industries. The application of the falling film principle has the advantage of short residence time which is most desirable for heat-sensitive materials. In short columns and when the viscosity of the solution is high, the film flow may be laminar in nature. Little information, however, is available on the heat transfer rate in these liquid films. Murthy and Sarma [1] investigated analytically heating in the entrance region of an accelerating, non-Newtonian, power-law-model, laminar falling film flowing down an inclined plane with constant wall temperature. Integral solutions for the boundary-layer equations of momentum and energy were obtained in which the Nusselt number for the thermally developing and fully developed regions can be calculated. Heating with constant wall temperature and a fully developed velocity profile was also analyzed both theoretically and experimentally by Stucheli and Widmer [2] for Newtonian and non-Newtonian power-law model falling film on an inclined plane. The viscosity was assumed to be temperature dependent. The objectives of the present research are to show that a simple analytical solution can be easily obtained for heating or evaporation in the thermal entrance and fully developed regions of a non-Newtonian, power-law model falling film with the boundary condition of constant wall heat flux or constant wall temperature.

2. THEORY

A non-Newtonian liquid film of average film thickness, δ, is in steady laminar flow down a vertical plane under the action of gravity. The liquid flow is characterized by a power-law rheological model. The velocity profile of the falling film is assumed to be fully developed at the start of the heat transfer section. By a balance of shear and gravity forces, the dimensionless velocity profile can be derived, with the boundary condition of no slip at the wall ($y = 0$) and zero interfacial shear at the gas–liquid interface ($y = \delta$), as

$$U^*(\eta) = U(\eta)/U_1 = 1-(1-\eta)^{(n+1)/n} \quad (1)$$

where $\eta = y/\delta$ and y is the distance measured from the wall into the liquid film with x being the coordinate in the flow direction. The average velocity and surface velocity can be derived as

$$U_{av}/U_1 = (n+1)/(2n+1) \quad (2)$$

$$U_1 = \frac{n}{n+1}\left(\frac{\rho g}{K}\right)^{1/n} \delta^{(n+1)/n}. \quad (3)$$

The film thickness δ is

$$\delta = \left\{\frac{[(2n+1)/n]Q}{B}\right\}^{n/(2n+1)} \bigg/ \left(\frac{\rho g}{K}\right)^{1/(2n+1)} \tag{4}$$

where Q/B is the flow rate per unit periphery, K is the consistency index and n is the power-law index.

2.1. *Heating with constant wall heat flux*

The energy equation is represented, in dimensionless form, by

$$U^*(\eta)\frac{\partial \theta}{\partial x^*} = \frac{\partial^2 \theta}{\partial \eta^2} \tag{5}$$

where θ is defined here as $\theta = (T - T_{\text{in}})/(q_w \delta/k)$, $x^* = x\alpha/\delta^2 U_1$. The boundary conditions are

$$x^* = 0 \quad \theta = 0 \tag{6}$$

$$\eta = 0 \quad \frac{\partial \theta}{\partial \eta} = -1 \tag{7}$$

$$\eta = 1 \quad \frac{\partial \theta}{\partial \eta} = 0. \tag{8}$$

Equations (5)–(8) can be reduced by linear superposition to two sets of equations [3, 4] by letting

$$\theta(x^*, \eta) = \theta_1(x^*, \eta) + \theta_2(x^*, \eta) \tag{9}$$

where θ_1 and θ_2 represent the entrance region and thermally developed solutions respectively. The solution of θ_2 can be obtained by a method similar to Yih and Liu's [5].

Thermally fully developed region. For film heating, when the velocity and temperature profiles are fully developed

$$\frac{\partial \theta_2}{\partial x^*} = \frac{d\theta_{2\text{m}}}{dx^*} = \frac{d\theta_{2\text{w}}}{dx^*} = \text{constant} \tag{10}$$

where θ_m and θ_w are, respectively, the dimensionless bulk average and wall temperatures. Equation (10) shows that θ_2 is of the form

$$\theta_2 = (\text{constant})x^* + F(\eta). \tag{11}$$

The set of equations and boundary conditions for θ_2 obtained by linear superposition are integrated to give

$$\theta_2 = \left(\frac{2n+1}{n+1}\right)x^* + \left(\frac{2n+1}{n+1}\right)\int_0^\eta \left[\int_0^\eta U^*\, d\eta - \left(\frac{n+1}{2n+1}\right)\right] d\eta. \tag{12}$$

It can be seen that $F(\eta)$ is the second term on the RHS of equation (12) and that

$$\theta_{2\text{w}} = \left(\frac{2n+1}{n+1}\right)x^* \tag{13}$$

$$\frac{d\theta_{2\text{m}}}{dx^*} = \frac{2n+1}{n+1}. \tag{14}$$

The local heat transfer coefficient for heating with constant wall heat flux is defined as $h_x = q_w/(T_w - T_m)$ and the Nusselt number is equal to $Nu_x = h_x \cdot \delta/k = 1/(\theta_w - \theta_m)$. The asymptotic Nusselt number can be derived as

$$Nu_\infty = \left\{\left(\frac{2n+1}{n+1}\right)^2 \int_0^1 U^* \left[\int_0^\eta \left(\frac{n+1}{2n+1} - \int_0^\eta U^*\, d\eta\right) d\eta\right] d\eta\right\}^{-1}. \tag{15}$$

Thermal entrance region. The set of equations and boundary conditions for θ_1 obtained by linear superposition can be solved by the method of separation of variables in a manner analogous to that described by Yih and Chen [4]. The result for Nu_x is

$$Nu_x = 1/(\theta_w - \theta_m) = \left(\frac{n+1}{2n+1}\right) \bigg/ \left[\int_0^1 U^*(\theta_w - \theta)\, d\eta\right] \tag{16}$$

where

$$\theta - \theta_w = \sum_{i=1}^\infty A_i N_i(\eta) \exp(-\lambda_i^2 x^*) + F(\eta) - \sum_{i=1}^\infty A_i \exp(-\lambda_i^2 x^*) \tag{17}$$

$$\theta_m - \theta_w = \int_0^1 U^*(\theta - \theta_w)\, d\eta \bigg/ \left(\frac{n+1}{2n+1}\right). \tag{18}$$

In equation (17), λ_i is the eigenvalue, N_i is the eigenfunction and A_i is the series coefficient as determined by

$$A_i = \int_0^1 U^*[-F(\eta)]N_i\, d\eta \bigg/ \int_0^1 U^* N_i^2\, d\eta. \tag{19}$$

2.2. *Heating with constant wall temperature*

The energy equation is the same as equation (5) where θ is now defined as $\theta = (T_w - T)/(T_w - T_{\text{in}})$. The boundary conditions are

$$x^* = 0, \quad \theta = 1 \tag{20}$$

$$\eta = 0, \quad \theta = 0 \tag{21}$$

$$\eta = 0, \quad \partial\theta/\partial\eta = 0. \tag{22}$$

The solutions can be obtained directly by the method of separation of variables [3, 4]. θ_m and Nu_x can be derived as

$$\theta_m = \left[\sum_{i=1}^\infty A_i \exp(-\lambda_i^2 x^*) N_i'(0)/\lambda_i^2\right] \bigg/ \left(\frac{n+1}{2n+1}\right) \tag{23}$$

$$Nu_x = \left[\sum_{i=1}^\infty A_i \exp(-\lambda_i^2 x^*) N_i'(0)\right] \bigg/ \theta_m \tag{24}$$

where the local heat transfer coefficient for heating with constant wall temperature is defined as

$$h_x = -k\frac{\partial T}{\partial y}\bigg|_{y=0} \bigg/ (T_w - T_m).$$

When $x^* \to \infty$ the asymptotic Nusselt number can be obtained from equations (23) and (24) by retaining only the first term in the summation series to give

$$Nu_\infty = \left(\frac{n+1}{2n+1}\right)\lambda_1^2 \tag{25}$$

2.3. *Evaporation with constant wall heat flux*

When the evaporation rate is not very large, the average film thickness can be regarded as constant. The energy equation, equation (5), still applies but θ is now defined as $\theta = (T - T_{\text{sat}})/(q_w \delta/k)$ and the boundary conditions are

$$x^* = 0, \quad \theta = 0 \tag{26}$$

$$\eta = 0, \quad \partial\theta/\partial\eta = -1 \tag{27}$$

$$\eta = 1, \quad \theta = 0. \tag{28}$$

Again we let $\theta = \theta_1 + \theta_2$ by linear superposition. For film evaporation, when the velocity and temperature profiles are fully developed, $\partial\theta_2/\partial x^* = 0$ and θ_2 can be obtained as

$$\theta_2 = \int_\eta^1 d\eta. \tag{29}$$

The local heat transfer coefficient for evaporation with constant wall heat flux is defined as $h_x = q_w/(T_w - T_{\text{sat}})$ and

$$Nu_\infty = 1/\theta_w = 1 \bigg/ \int_0^1 d\eta = 1 \tag{30}$$

θ_1 is solved by separation of variables and the series coefficient A_i is determined by

$$A_i = \int_0^1 U^*(-\theta_2)N_i\, d\eta \bigg/ \int_0^1 U^* N_i^2\, d\eta. \tag{31}$$

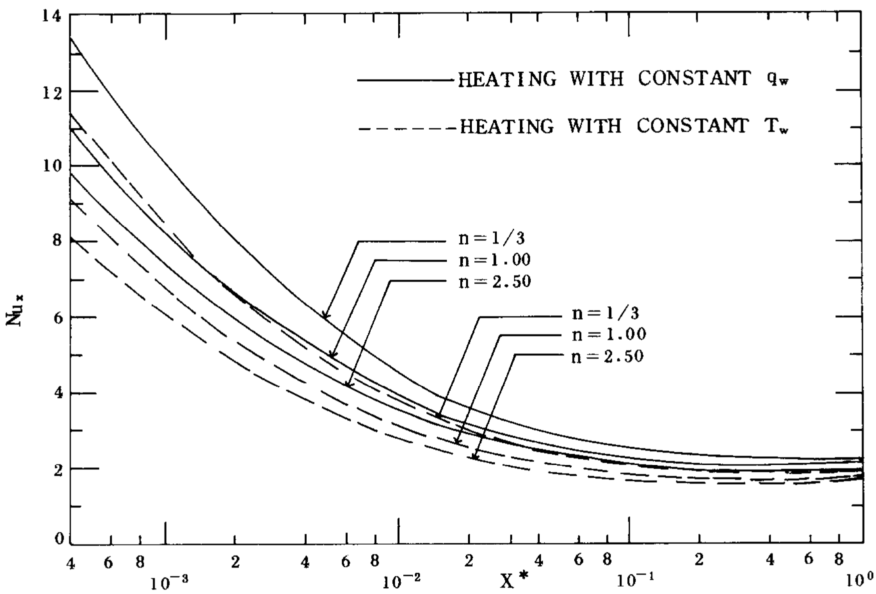

FIG. 1. Comparison of local Nusselt number for heating with constant wall heat flux and constant wall temperature.

The local Nusselt number is

$$Nu_x = \left[\sum_{i=1}^{\infty} A_i N_i(0) \exp(-\lambda_i^2 x^*) + 1\right]^{-1}. \quad (32)$$

2.4. *Evaporation with constant wall temperature*

Equation (5) still applies but θ is now defined as $\theta = (T_w - T)/(T_w - T_{sat})$. The boundary conditions are

$$x^* = 0, \quad \theta = 1 \quad (33)$$

$$\eta = 0, \quad \theta = 0 \quad (34)$$

$$\eta = 1, \quad \theta = 1. \quad (35)$$

We again let $\theta = \theta_1 + \theta_2$, θ_2 and Nu_∞ are solved to give

$$\theta_2 = \int_0^\eta d\eta \quad (36)$$

$$Nu_\infty = \left.\frac{\partial \theta_2}{\partial \eta}\right|_{\eta=0} = \int_0^1 d\eta = 1 \quad (37)$$

θ_1 is solved also to give A_i and Nu_x as

$$A_i = \int_0^1 (1-\theta_2) U^* N_i \, d\eta \Big/ \int_0^1 U^* N_i^2 \, d\eta \quad (38)$$

$$Nu_x = \sum_{i=1}^{\infty} A_i \exp(-\lambda_i^2 x^*) N_i'(0) + 1. \quad (39)$$

3. RESULTS AND DISCUSSION

Extensive numerical results of Nu_x, the first 15 eigenvalues, series coefficients and associated quantities are obtained for various values of the power-law index of n = 1/3, 0.5, 1.0, 1.25, 2.5 covering a range of practical interest. For short contact times of heat transfer, the Leveque solution is valid for either heating or evaporation,

$$Nu_x = 0.538\left(\frac{n+1}{n}\right)^{1/3} (x^*)^{-1/3} \quad \text{for constant } T_w \quad (40)$$

$$Nu_x = 0.651\left(\frac{n+1}{n}\right)^{1/3} (x^*)^{-1/3} \quad \text{for constant } q_w. \quad (41)$$

Heating with constant wall temperature corresponds to the problem of solid dissolution in a falling film and has been studied by Mashelkar and Chavan [6], and Yih and Huang [3] who presented the same eigenvalues. Figure 1 gives a comparison of Nu_x for the two different boundary conditions. At a given n, the Nusselt number for heating with constant wall heat flux is always larger than that for heating with constant wall temperature. However, heating with constant wall temperature generates a shorter thermal entrance length than heating with constant wall heat flux. For a fixed x^*, Nu_x increases with decreasing values of n. However, the thermal entrance length also increases with decreasing values of n. The first 10 eigenvalues for evaporation with constant wall temperature are shown in Table 1. A comparison of heating and evaporation is shown in Fig. 2. At very small x^*, the Nusselt number for heating and evaporation are the same and conform to the Leveque solution. However, with increasing x^*, the Nusselt number for heating becomes larger than that for evaporation. Also the thermal entrance length for heating is always shorter than that for evaporation. A direct application of this work would be to devise a new method for measuring the thermal conductivity of dilute polymeric solutions. If the variation of viscosity with temperature is important as examined by Stucheli and Widmer [2], the present method of solution can easily incorporate this effect too [7].

Table 1. First 10 eigenvalues for evaporation with constant wall temperature

	n				
i	2.50	1.25	1.00	0.50	1/3
1	3.97257	3.75093	3.67232	3.43818	3.32752
2	8.28298	7.83153	7.66884	7.16549	6.90498
3	12.5997	11.9155	11.6679	10.8968	10.4938
4	16.9178	16.0001	15.6675	14.6295	14.0855
5	21.2365	20.0849	19.6673	18.3628	17.6784
6	25.5553	24.1698	23.6672	22.0964	21.2717
7	29.8742	28.2547	27.6671	25.8302	24.8654
8	34.1932	32.3397	31.6671	29.5641	28.4594
9	38.5123	36.4248	35.6671	33.2982	32.0534
10	42.8315	40.5099	39.6672	37.0323	35.6476

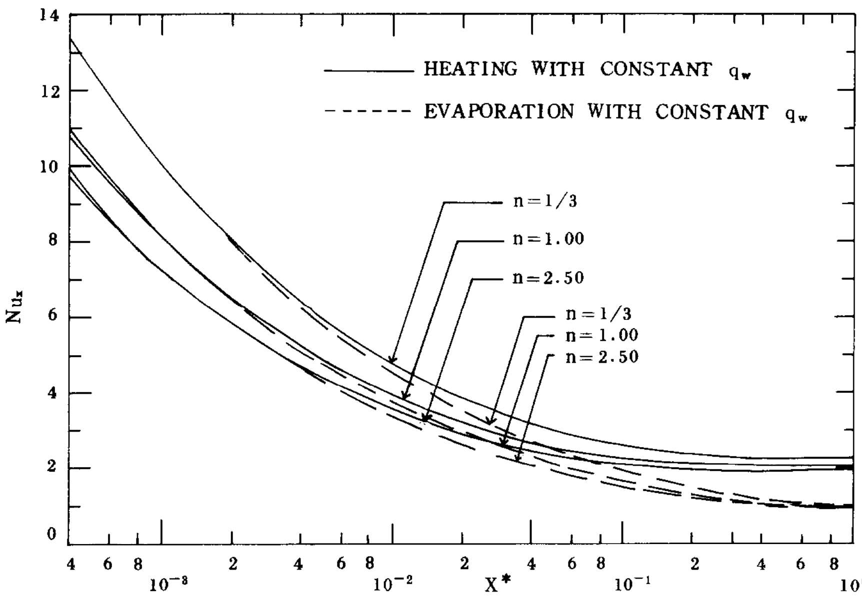

FIG. 2. Comparison of local Nusselt number for heating and evaporation with constant wall heat flux.

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Int. J. Heat Mass Transfer. Vol. 29, No. 12, pp. 2002–2006, 1986
Printed in Great Britain
0017–9310/86 $3.00+0.00
Pergamon Journals Ltd.

Some aspects of enhanced heat diffusion in fluids by oscillation

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(*Received* 10 *December* 1985 *and in final form* 11 *March* 1986)

1. INTRODUCTION

THE SUBJECT of enhanced heat transfer (passive or active) can be divided into two areas: (i) improvement of heat transfer rates at solid surface–fluid interfaces; and (ii) improvement in transport capabilities of the conducting fluid. In the former case it has been shown theoretically and experimentally that the oscillation of the surface can increase the heat transfer rate, and various geometries and techniques have been explored [1–3]. In the latter case a variety of methods have been considered, including wicked or gravity-driven heat pipes and packed-bed heat pipes [4]. Recently the laminar oscillation of an otherwise stationary fluid has also been studied [5–11]. The oscillation technique, which is the subject of this study, is based on the periodic longitudinal convective–lateral diffusive thermal energy transport in the presence of a longitudinal temperature gradient. This can result in a very significant increase in the longitudinal transport capability of the fluid.

This idea was initially applied to the enhancement of longitudinal oxygen dispersion in pulsating flows in pulmonary systems [5, 6, 8] and has recently been applied to axial transport of thermal energy [9–11].

The available studies of this phenomenon, which are based on the application of the conservation equations to periodic laminar flows, have led to the prediction of the longitudinal heat transfer rates [10, 11]. Closed-form solutions have been found for one-dimensional and linear velocity and temperature fields. Experimentally [9], enhanced heat transfer rates have been found using water as the fluid, 1-mm-ID capillary tubes, oscillation frequencies between 2 and 8 Hz and tidal displacements (i.e. the average particle dis-